Deformed stress-tensor multiplets and supersymmetry algebras

Stress-tensor multiplet:
- stress-tensor $T_{\mu\nu}$
- N_Q supersymmetry currents $S^i_{\mu\alpha}$
- R-symmetry currents

$\rightarrow$ trace-less at conformal point:

$$T^\mu{}_\mu = 0, \qquad \sigma^{\mu\dot\alpha\alpha} S^i_{\mu\alpha} = 0 \qquad \text{(4d notation)}$$

deformation $\delta\mathcal{L} = \lambda\,\mathcal{O}$ leads to

$$T^\mu{}_\mu \sim \lambda(\Delta - d)\,\mathcal{O} + \mathcal{O}(\lambda^2)$$

only preserves conformal symmetry if $\mathcal{O}$ is marginal.

otherwise: superconformal sym. $\rightarrow$ Poincaré super-sym.

example:

4d $\mathcal{N}=1$ SCFTs

$T_{\mu\nu} \in A_1\bar{A}_1[0;0]^{(0)}_2$,

primary: $\mathcal{J}_\mu = \bar\sigma^{\alpha\dot\alpha}_\mu \mathcal{J}_{\alpha\dot\alpha}$ $\quad U(1)_R$ current

shortening conditions: $\bar{Q}^{\dot\alpha}\mathcal{J}_{\alpha\dot\alpha} = Q^\alpha \mathcal{J}_{\alpha\dot\alpha} = 0$

$\rightarrow$ 8 + 8 bosonic and fermionic operators!

- conserved R-current $\mathcal{J}_\mu$
- conserved, traceless supersymmetry currents $S_{\mu\alpha}$, $\bar{S}_{\mu\dot{\alpha}}$
- stress-tensor $T_{\mu\nu}$

Poincaré SUSY:

shortening condition $\xrightarrow{\text{deform.}}$

$$\bar{Q}^{\dot{\alpha}} \mathcal{J}_{\alpha\dot{\alpha}} = Q_\alpha X \, , \quad Q^\alpha \mathcal{J}_{\alpha\dot{\alpha}} = \bar{Q}_{\dot{\alpha}} \bar{X} \, , \quad \bar{Q}_{\dot{\alpha}} X = Q_\alpha \bar{X} = 0$$

chiral sub-multiplet

$$\rightarrow \begin{bmatrix} 8+8 \\ \text{(SCM)} \end{bmatrix} + \begin{bmatrix} 4+4 \\ \text{(X-mult.)} \end{bmatrix} = \begin{bmatrix} 12+12 \\ \text{Poincaré mult.} \end{bmatrix}$$

"Ferrara-Zumino" multiplet

$\mathcal{J}_\mu$ is not conserved any longer

<u>flavor mass deformation</u>:

$N_Q = 8$ theories can be obtained from 6d $\mathcal{N} = (1,0)$ theories

$\mathcal{J}_a^{\,ij}$ (flavor current primary in 6d)

adjoint flavor index

$SU(2)_R$ doublet indices

$\rightarrow$ reduction on S^1, T^2, T^3 leads to flavor current multiplets with 0, 1, 2, 3 scalar mass def. in $d = 6, 5, 4, 3$

Primaries and shortening conditions:

$$Q_\alpha^{(i} J_a^{jk)} = 0 , \qquad Q_\alpha^{(i} J_a^{jk} = \bar{Q}_{\dot\alpha}^{(i} J_a^{jk)} = 0$$

$d=5$ $\qquad\qquad\qquad\qquad$ $d=4$

$$Q_\alpha^{i'(i} J_a^{jk)} = 0 \quad \text{or} \quad Q_\alpha^{i(i'} J_\alpha^{j'k')} = 0$$

$d=3$

where $\alpha, \dot\alpha$ are space-time spinor indices and i, j, k are $SU(2)_R$ doublet indices.

- stress-tensor multiplets:

  L- and R-sym. singlet T is primary with scaling dimension $\Delta = d-2$

  shortening conditions:

$$\Omega^{\alpha\beta} Q_\alpha^{(i} Q_\beta^{j)} T = 0 , \qquad \varepsilon^{\alpha\beta} Q_\alpha^{(i} Q_\beta^{j)} T = \varepsilon^{\dot\alpha\dot\beta} \bar{Q}_{\dot\alpha}^{(i} \bar{Q}_{\dot\beta}^{j)} T = 0$$

$d=5$ $\qquad\qquad\qquad\qquad$ $d=4$

$$\varepsilon^{\alpha\beta} Q_\alpha^{(i'i} Q_\beta^{j'j)} T = 0$$

$d=3$

$\longrightarrow$ $\underbrace{8(d-1)}_{\text{bosonic}} + \underbrace{8(d-1)}_{\text{fermionic}}$ operators

$\Big\downarrow$ $d \to d-1$ reduction

$\underbrace{\left(8(d-2) + 8(d-2)\right)}_{d-1\text{-dim stress-tensor}} + \underbrace{(8+8)}_{\text{flavor mult.}}$

where flavor-mult. is associated to KK symmetry in the reduction

→ non-conformal stress-tensor mult. :

flavor mass-def.

$$\delta \mathcal{L}_{fl.} = m_{a,I} \mathcal{U}^{a,I}_{fl.} + \mathcal{O}(m^2)$$

$$= m_{a,I} \left(Q^2 \mathcal{J}^a\right)^I + \mathcal{O}(m^2)$$

can be viewed as Wilson lines that wrap the reduced dimensions

$d=5$: $\Omega^{\alpha\beta} Q^{(i}_\alpha Q^{j)}_\beta \mathcal{T} = m_a \mathcal{J}_a^{ij}$,

$d=4$: $\varepsilon^{\alpha\beta} Q^{(i}_\alpha Q^{j)}_\beta \mathcal{T} = m_a \mathcal{J}_a^{ij}$,

$$\varepsilon^{\dot\alpha\dot\beta} \bar{Q}_{\dot\alpha}^{(i} \bar{Q}_{\dot\beta}^{j)} \mathcal{T} = \bar{m}_a \mathcal{J}_a^{ij}$$

$d=3$: $\varepsilon^{\alpha\beta} Q^{(i'i}_\alpha Q^{j'j)}_\alpha \mathcal{T} = m_a^{i'j'} \mathcal{J}_a^{ij} + (m')^{ij}_{a'} \mathcal{J}_{a'}^{i'j'}$

conserved flavor currents on the right side integrate to Lorentz-scalar central charges :

$$\Omega_{\alpha\beta} \varepsilon^{ij} Z \subset \left\{ Q^i_\alpha , Q^j_\beta \right\}$$

where $Z = m_a F_a$

masses flavor charges

universal mass deformation:

3d $\mathcal{N} \geq 4$ SCFTs

univ. mass def. $\subset$ stress-tensor mult.

example: $\mathcal{N}=4$

m preserves entire $SU(2)_R \times SU(2)_R'$ symmetry with generators R^{ij}, $(R')^{ij}$

$$\rightarrow \{Q_\alpha^{i'i}, Q_\beta^{j'j}\} = \varepsilon^{ij}\varepsilon^{i'j'}P_{(\alpha\beta)} + 2m\,\varepsilon_{\alpha\beta}\varepsilon^{i'j'}R^{ij} - 2m\,\varepsilon_{\alpha\beta}\,\varepsilon^{ij}(R')^{i'j'} \quad (*)$$

R-sym. generators are non-central!

$\rightarrow$ theory is gapped:

consider a mass-less particle with lightcone momentum $P_+ = E$, $P_- = P_3 = 0$

$\rightarrow (*)$ reduces to:

a) $\{Q_+^{i'i}, Q_+^{j'j}\} = \varepsilon^{ij}\varepsilon^{i'j'}E$,

b) $\{Q_-^{i'i}, Q_-^{j'j}\} = 0$

c) $\{Q_+^{i'i}, Q_-^{j'j}\} = 2m\varepsilon^{i'j'}R^{ij} - 2m\,\varepsilon^{ij}(R')^{i'j'}$

b) $\Rightarrow Q_- = 0$

c) $\Rightarrow R^{ij}$ and $(R')^{i'j'}$ act trivially

$\Rightarrow Q_+ = 0$

a) $\Rightarrow E = 0$ (theory is gapped)